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Dynamical systems techniques were being developed and applied to problems of transport in fluid flows, e.g., turbulent heat transfer. Earlier successful development for two-dimensional flows were extended (1) to include the effect of diffusion in the presence of a wall, (2) to investigate steady and unsteady three-dimensional effects, and (3) to study interface dynamics.

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New Predictive Methods in Chaotic Fluid Dynamics with Application to Turbulent Heat Transfer

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FINAL REPORT

New Predictive Methods in Chaotic Fluid Dynamics with Application to Turbulent Heat Transfer

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Introduction

The transport of "passive scalars", e.g., heat or species concentrations in fluids was investigated. Convective transport and mixing in fluids are closely tied to the unsteady global structure of the flow, i.e., its topological features and dynamics, and these play an important role in temperature/concentration distributions, residence-times and fluxes. Questions pertaining to transport in fluid flows can be examined from a dynamical system viewpoint since the problem can be viewed in the more general context of transport in phase space. The questions of interest then involve an understanding of the global dynamics in the phase space of the dynamical system and the role played by geometrical structures such as invariant manifolds in the advection of passive scalars by the velocity field.

Dynamical systems techniques for studying transport without diffusion effect in two-dimensional, time-periodic or quasi-periodic, flows had already been developed by the present investigators. To develop further understanding and to make quantitative productions for realistic flows the following were to be studied: (1) effects of diffusion in the presence of a wall and (2) effects of fluid turbulence including interface stretching. In addition complementary studies were undertaken including (3) the analysis of transport and diffusion in a specific time-periodic 3D flow and (4) the development of analytical methods for studying particle paths in a class of three-dimensional flows.

I. Advection-Diffusion in Chaotic Flows

The kinematics of a perfect or non-diffusive tracer is purely advective. In a laminar flow, perfect tracer particles will follow the pathlines of the flow. The dispersion of these particles is then directly related to the fluid particle trajectories and, in Eckart's[1] terminology, is called "stirring". Given some initial distribution of perfect tracer particles, how this distribution evolves in time is entirely dependent on the dynamics of particle motion in the flow, and therefore the associated transport issues are often best understood using the global geometrical viewpoint of dynamical systems theory. In stirring by chaotic advection[2], the individual particle trajectories might be very complicated, but the underlying geometrical structures such as invariant manifolds and homoclinic/heteroclinic tangles provide a dynamical template that in certain cases considerably simplifies questions related to the transport or dispersion of particles (see, e.g. Wiggins[3]). A scalar impurity will, however, undergo both advection and diffusion. Thus the time-evolution of some given initial scalar field will be dictated not only by the purely fluid-mechanical stirring process but also by the generally slower process of molecular diffusion of the now diffusive tracer, which is called "mixing"[1]. In the Lagrangian framework, the kinematics of a diffusive tracer has a Brownian-motion-component in addition to the advective component due to the fluid motion, and tracer particles no longer follow the pathlines of the flow. That raises several fundamental questions regarding the role of the underlying geometrical structures in the transport.

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An important isssue which has been mostly ignored in the existing literature is the transport of a passive scalar from an active no-slip boundary into a chaotic flow, even though heat and mass transfer from stationary surfaces is common in engineering applications. The linear part of the velocity field expanded about any stagnation point on the no-slip boundary has zero eigen-values, and therefore every point on the no-slip boundary is non-hyperbolic. The non-hyperbolicity of the stagnation points on a no-slip boundary makes analysis difficult. Further, stirring, by itself, becomes meaningless since diffusion is essential for "lifting" heat or a passive impurity from the active no-slip surface. Given these complications, it is not clear how the geometrical structures in a chaotic flow over a active no-slip surface can influence the time-dependent distribution of the scalar field as the scalar impurity diffuses into the flow. Therefore, one objective of the study was to investigate some of these issues using the model flow of a two-dimensional time-periodic separation bubble with chaotic particle trajectories, over a plane stationary surface.

A method devised by Perry and Chong[6] was used to obtain a simple Taylor-series representation of a chaotic separation bubble which is also an asymptotically exact solution of the Navier-Stokes and continuity equations, close to the origin of the series-expansion. The method relies on the availability of sufficient number of topological constraints[6] and is therefore particularly well-suited to study steady two- and three-dimensional separated flows[6,7] on account of their readily available topological features such as location and stability-type of stagnation points, location of points of zero shear-stress on the no-slip boundary, angles of separation and attachment, etc. The scheme was to construct a low-order series-representation of a steady two-dimensional separation bubble at a plane wall and then introduce time-periodic terms to obtain an unsteady bubble with chaotic particle trajectories, such that the representation satisfies incompressibility and remains an asymptotically exact solution of the Navier-Stokes equations.

The relative importance of advection versus diffusion is measured by the Peclet number, which is the ratio of the advection and diffusion time-scales. Of special interest is the regime of small scalar diffusion, or more precisely, the regime where the diffusion time-scale is much greater than the advection time-scale, which means large Peclet numbers. At large Peclet numbers, the scalar advection-diffusion problem is best tackled by random-walk methods based on the theory of Brownian motion[8], and numerical implementations of these methods were developed to solve for the time-evolution of the scalar field. A fictitous "zero-diffusivity" solution was introduced as a heuristic tool in demonstrating the role of the underlying geometrical structure in the flow and in interpreting the role of slow mixing as a local smoothing of fine-scale structure in the scalar field, created by the stirring process.

An invariant hyperbolic set[9] is the prototype of a chaotic dynamical system, and the *shadowing lemma*[10] from dynamical systems theory is one of the fundamental results for the dynamics on an invariant hyperbolic set. Recent work of Klapper[11] has used shadowing theory[10,12] to study the small-diffusivity scalar advection-diffusion problem. Asymptotic results were obtained for the restricted class of uniformly hyperbolic systems, and therefore apply to typical chaotic processes in only a non-rigorous sense. Justification[11] for its validity is based on existing numerical evidence[13–15] that typical chaotic dynamical processes have the shadowing property. In a rough sense, a dynamical system that has the shadowing property is guaranteed to have a deterministic orbit that remains close to any noisy orbit with bounded noise, where how "close" depends on the noise level. The shadowing property has been used previously to reduce bounded additive noise in orbits generated by chaotic dynamical systems[16,17]. That the shadowing property can be used to treat scalar diffusion is not surprising since diffusion can be regarded as a noisy component in the kinematics of a diffusive tracer. These ideas were used to develop a better understanding of random-walk solutions of the time-dependent scalar field and the interplay between the stirring and mixing processes. One of the surprising results of the present work is that increased chaotic advection does not necessarily promote

mixing and can produce more localized and non-uniform distributions, even in regions of the flow that have no islands of stability bounded by invariant closed curves; near integrability, such curves will be provided by Kolmogorov-Arnold-Moser (KAM) tori and island bands, but as one perturbs the dynamical system further away from integrability there are no surviving invariant closed curves, and such a parametric regime was chosen to emphasize the result. Scalar field computations are also carried out for the set of parameters at which KAM tori and island bands occupy a significant portion of the flow region above the plane active surface. The time-evolution of the scalar field in this case bears interesting resemblance to that for the steady separation-bubble-flow. Also examined were the asymptotic form of the scalar field as well as the integrated wall-flux. See the publication by Ghosh, Leonard, and Wiggins (attached) for more information.

II. Studies of Particle Motions and Similarity Laws in Decaying Homogeneous Turbulence

Direct numerical simulations of decaying homogeneous turbulence were performed, according to the incompressible Navier-Stokes equations, on the Intel Delta Parallel Machine at Caltech using Fourier pseudo-spectral methods. Spectral meshes up to and including 512^3 were used.

Power law decay in the turbulent energy was observed with exponent about $\frac{3}{2}$ in some cases and about $\frac{6}{5}$ in other cases. This led to the reexamination of the basis for self-similar forms for the velocity correlation functions in the Karman Howarth equation. The equation can be closed if the triple correlation tensor in it is negligible (Von Karman and Howarth 1938) or some kind of separability constraint is invoked (Sedov 1994, 1959; Speziale 1992). A new similarity law was proposed in which three functions are required to define the double and triple correlation tensors for isotropic turbulence. One of these functions is determined explicitly with a continuous spectrum consisting of the possible decay exponents. The third function is given in terms of the second. If Saffman's invariant (Saffman 1963) is recalled, the decay exponent resulting from the new similarity law agrees with $\frac{3}{2}$. See the publication by Huang and Leonard (attached) for more details.

Lagrangian and Eulerian double velocity correlation tensors have been measured also. The limitation on the similarity of the Eulerian correlation tensors of the numerical turbulence was studied. Similarity of the Lagrangian correlations was investigated and computed scaled by Sato and Yamomoto's time variable (1987). Particle diffusion is measured and compared with the results of the similarity theory of Lagrangian velocity double correlation tensors.

The history of growth rates of material line elements was also studied along with that of the principal rates of the strain following a fluid particle. Particle tracing methods were developed based on the work of Yeung and Pope (1988, 1989; see also Girimaji and Pope 1990). Of particular interest was the sudden intensification of vorticity seen to occur at some particle locations which usually lasts for about one Lagrangian integral time. A careful examination on the flows has been done to study the correlation among the evolutions of small-scale vorticity structures and particle locations. The growth rates of line elements and the principal rates of the strain tensors when normalized by the temporal mean enstrophy were found approximately constant during the decay. The ratios of the principal rates were compared with those of physical models of turbulence. Lundgren's spiral vortex model (Lundgren 1982; Pullin and Saffman 1993) appears to offer the best match.

Relationships among vorticity vectors, material line elements and the principal directions of the strain tensors were investigated also. These relationships were examined particularly when the intensification of vorticity occurs so as to provide an understanding of the small-scale vorticity structures. The effect of viscosity on the alignment of vorticity vectors has been studied by comparing them with those of material line elements. Differences have been observed and discussed.

III. Chaotic Transport and Dispersion Near a Helical Vortex Filament in a Time-Periodic Potential Flow: An Analytical and Numerical Study

The flow induced by a helical vortex filament in a three-dimensional time-dependent strain field was analyzed. The equations of motion for a vortex filament moving under the Localized Induction Approximation in an external potential flow were derived. It was shown that one solution of these is the case of a helical vortex filament in a three-dimensional time-dependent strain field. The filament moves in such a way that it stays helical for all times. Using symmetry concepts the velocity field was transformed to a particularly simple form. Bifurcations and the structure of particle paths were analyzed for the unperturbed velocity field, induced by the helix in the absence of the potential flow. The underlying geometrical structures in the unperturbed problem are cylinders and two dimensional separatrices. Away from separatrices the system was transformed to action-angle-angle coordinates. Using these coordinates a KAM-type theory for the persistence of cylinders, and a three-dimensional Melnikov theory for the analysis of the motion near the separatrices was described. In the case of the flow that was considered, the perturbed flow has the same symmetry as the unperturbed flow, so the correspondence between the symmetric perturbation and a nonsymmetric perturbation was discussed. Finally, the problem of shear dispersion in a class of flows admitting a symmetry was analyzed. Birkhoff's Ergodic Theorem was used to show under which conditions conjectures on shear dispersion through chaotic advection from some other works hold. The analysis was supported with numerical simulations.

IV. On the Integrability and Perturbation of Three-Dimensional Fluid Flows with Symmetry

Analytical methods for studying *particle paths* in a class of three-dimensional incompressible fluid flows were also developed. This was begun by the study of three-dimensional *volume preserving* vector fields that are invariant under the action of a one-parameter symmetry group whose infinitesimal generator is autonomous and volume preserving. It was shown that there exists a coordinate system in which the vector field assumes a simple form. In particular, the evolution of two of the coordinates is governed by a time-dependent, one-degree-of-freedom Hamiltonian system with the evolution of the remaining coordinate being governed by a first order differential equation that depends only on the other two coordinates and time. The new coordinates depend only on the symmetry group of the vector field. Therefore they are *field independent*. The coordinate transformation is constructive. If the vector field is time independent, then it posseses an integral of motion. Moreover, it was shown that the system can be further reduced to *action-angle-angle* coordinates. These are analogous to the familiar action-angle variables from Hamiltonian mechanics and should be quite useful for perturbative studies of the class of sytems we consider. In fact, it was also shown how this coordinate transformation puts one in a position to apply recent extensions of the KAM theorem for three-dimensional, volume-preserving maps as well as three-dimensional versions of Melnikov's method.

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The following doctoral students have participated in the effort during the grant period:

S. Ghosh: Graduate Research Assistant, Chemical Engineering
M. J. Huang: Graduate Research Assistant, Mechanical Engineering
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Publications Resulting From Grant Activities

1. Statistical Relaxation under Non-Turbulent Chaotic Flows: Non-Gaussian High-Stretch Tails of Finite-Time Lyapunov Exponent Distributions, D. Beigie, A. Leonard, and S. Wiggins, Phys. Rev. Lett. **70**, 275,18 January 1993.

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7. Chaotic Transport and Dispersion Near a Helical Vortex Filament in a Time-Periodic Strain Rate Field: An Analytical and Numerical Study, I. Mezic, A. Leonard, and S. Wiggins, submitted to J. Fluid Mech., 1994.

8. Birkhoff's Ergodic Theorem and Statistical Properties of Chaotic Dynamical Systems, with Applications to Fluid Mechanical Mixing and Dispersion, I. Mezic and S. Wiggins, submitted to Physica D, 1994.

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